Software Data T18 - STDDEV

T393 Issue 4 09/89

Please incorporate the following amendments:-

Page 1. OPERATING MODES

delete 'Quoted execution.....floating point maths.'

Page 3, Table 2

In the Range column opposite Sym N

delete +32767
insert +65535

Page 4, Table 3

add at end of table.

N = 65535	Q = Q_n - 1	9	0.3

Section 6

insert before 6.1

If the number of times a sample is taken exceeds 65535, N remains at 65535, the standard Deviation remains at the value calculated for N = 65535 and bits 0 and 3 in Zm are set.

Page 5, Section 6.1.2, line 5

delete Zm + 10
insert Zm + 8

Page 6

delete existing ladder diagram
insert new ladder diagram attached

Page 7, Section 7

delete

$$\delta^2 = \frac{(x_i - \bar{x})^2}{n-1}$$

insert

$$\delta^2 = \sum_{i=0}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

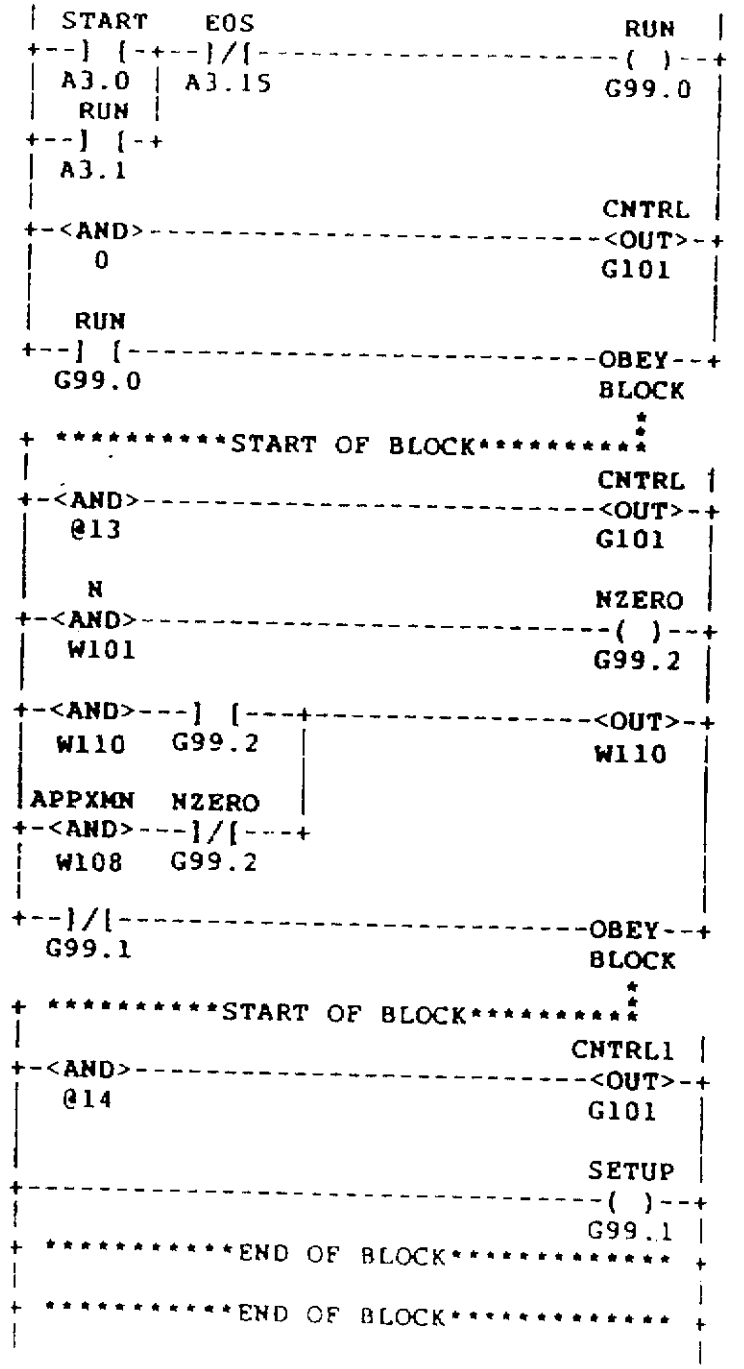
delete

$x = \Sigma x_i/n$ reduces to:

insert

$\overline{x} = \Sigma x_i/n$ reduces to:

Please retain this Errata Sheet with the document for reference.


START
A3.0
RUN
A3.1
EOS
A3.15
RUN
G99.0
AND
0
CNTRL
OUT
G101
RUN
G99.0
OBEY
BLOCK
**********START OF BLOCK**********
AND
@13
CNTRL
OUT
G101
N
AND
W101
NZERO
G99.2
AND
W110
G99.2
OUT
W110
APPXMN
AND
W108
NZERO
G99.2
G99.1
OBEY
BLOCK
**********START OF BLOCK**********
AND
@14
CNTRL1
OUT
G101
SETUP
G99.1
**********END OF BLOCK**********
**********END OF BLOCK**********
XI
AND
G100
STDDEV
SPEC
T18
VALUE
W100
STNDVN
OUT
W111
CNTRL
AND
G101

Mean and Standard Deviation

1. Description

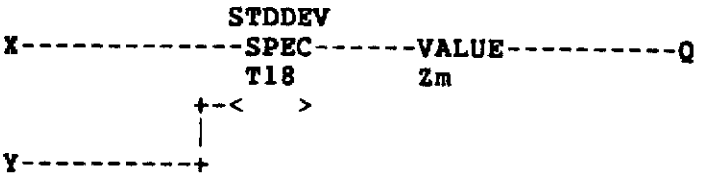
The STDDEV function calculates the mean and standard deviation of a number of values.

2. Features

- Floating point arithmetic ensures good accuracy and allows numbers up to 9E18 (9 x 10 to power 18).
- Calculation of mean and standard deviation can be "continuous" after each new samples, set by user.
- Use of approximate mean input reduces the range of values stored for calculation and thus improves accuracy.

3. Specification

Control Language:-



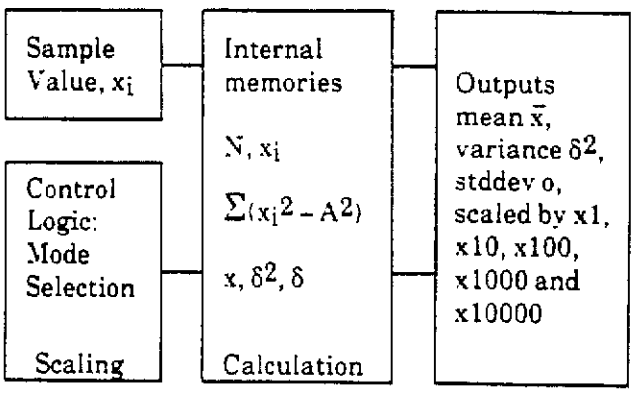


Table 1 Program rung data

Sym	Location	Use	Range
X	Upper rung input	Sample value, XI input when ACQUIRE mode selected	-32767 to +32767
Y	Lower rung input	Mode control input, CNTRL. Bit Function 0 ACQUIRE 1 CALCULATE 2 RESET 4 x10choose 5 x100scaling 6 x1000 7 x10000	Bits 0 to 2 ON to select mode. One of bits 4 to 7 ON to choose scaling of std deviation
Zm	Value	Start of value location table Zm) .) eleven .) locations Zm + 10)	Any write enabled table usable by special function
Q	Rung Output	Standard deviation Calculate value scaled by bits 4 to 7 of Y input.	0 to +32767

4. Operating modes

If desired, ACQUIRE and CALCULATE modes may be selected simultaneously.

Quoted execution times apply to 250 series controllers, using software floating point maths.

See last page for theory.

4.1 ACQUIRE Mode (bit 0 ON)

Input sample value Xi, add to the accumulated sum of sample values, square Xi and subtract approx. Mean squared, add to the accumulated sum of the same.

If N = 0 then take it , increment N.

Execution times approx. 2 ms.

4.2 CALCULATE Mode, (bit 1 ON)

Calculate mean, variance and standard deviation.

Execution times approx. 6ms.

4.3 RESET Mode (bit 2 ON)

Re-set accumulating memories to zero. This mode overrides ACQUIRE and CALCULATE.

Execution times 1 ms

5. Scaling

The output from the STDDEV function is an integer in the range 0 to 32767, scaled to represent a decimal value of standard deviation.

Bit On	Scaling Factor	Range for Standard Deviation
4	x10	0 to 3276.7
5	x100	0 to 327.67
6	x1000	0 to 32.767
7	x10000	0 to 3.2767

Table 2 Value data

Pairs of locations are used to store values in floating point form. It is not meaningful to monitor contents of those locations as decimals integers.

Sym	Location	Use	Range
F	Zm	Fault Code	See table 3
N	Zm + 1	Number of sample values acquired.	0 to +32767
Σx_i	Zm + 2 Zm + 3	Sum of sample Values acquired.	-9E18 to 9E18
$\Sigma(x_i^2-A^2)$	Zm + 4 Zm + 5	Sum of (square of Sample value minus approx mean squared)	0 to 9E18
δ^2	Zm + 6 Zm + 7	Variance.	0 to 9E18
A	Zm + 8	Approx mean input. Set by user in the event of a low sample count. **Note...** When this value is squared, it occupies two adjacent bits as follows.	-32767 to +32767
A^2	Zm + 8 Zm + 9	Approx mean squared	0 to 9E18
$\bar{x}$	Zm + 10	Mean.	-32767 to +32767

Table 3 Fault codes

Condition	Rung Output	Code in Numeric Value	Zm States ON
Invalid control input value Any of bits 8 to 15 On in Y.	Q = 0	5	0,2
No mode selected Y bits 0-2 OFF.	Q = X	0	None

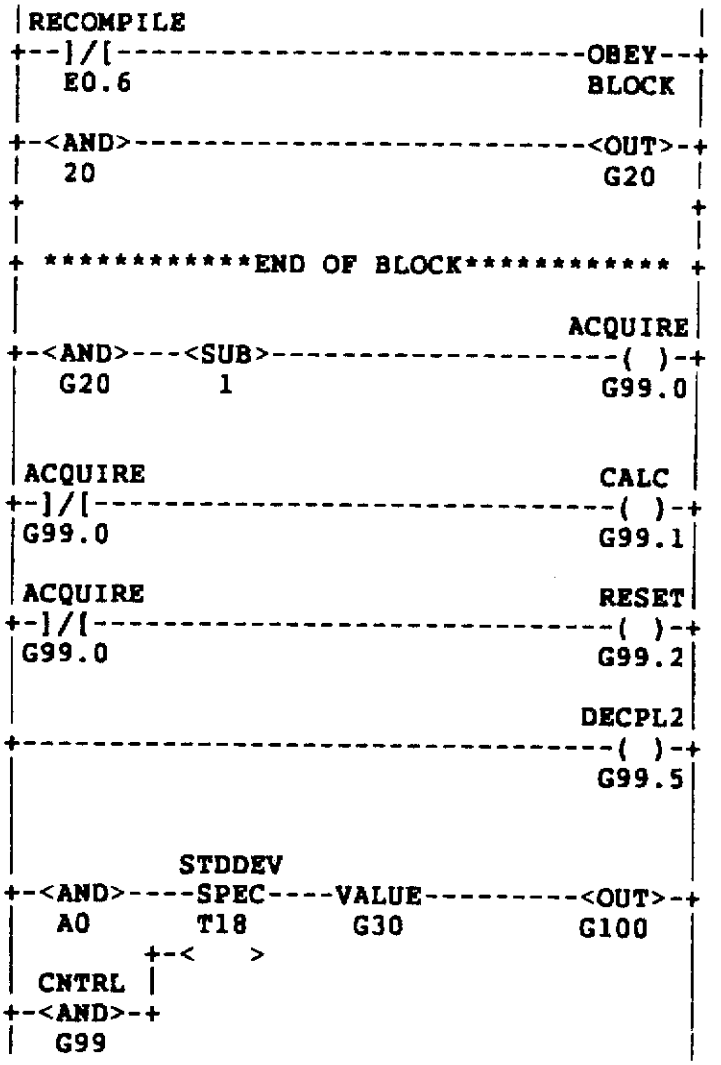
6. Application Notes

6.1 Program Examples

6.1.1 Batch Calculation

A batch of 20 values are to be sampled after which their mean and standard deviation are to be calculated and the summing register reset. The standard deviation value is to be scaled x100 representing 2 decimal places. No value of approximate mean is available.

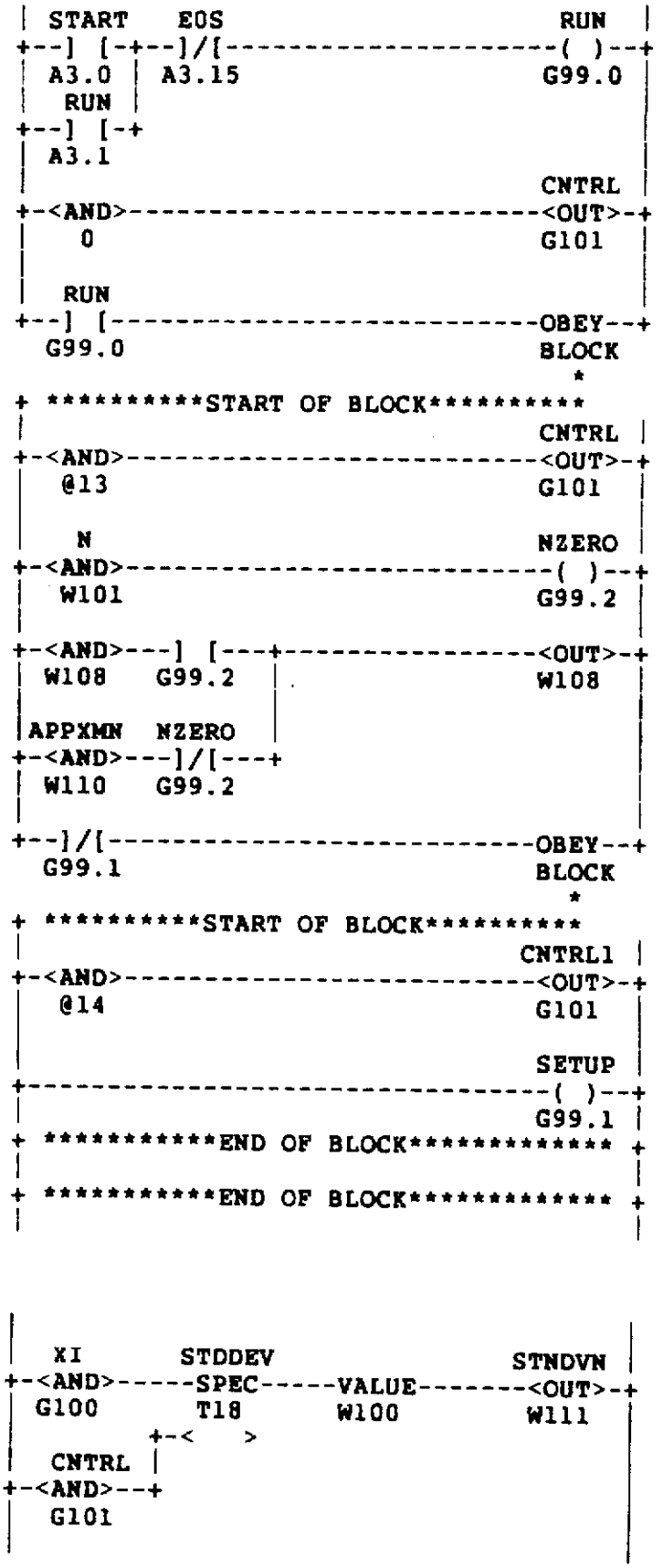
The following control rungs are a possible implementation of the above specification.



6.1.2 Continuous Calculation of Mean and Standard Deviation

Continuously calculate the mean and standard deviation of a series of values of one production shift. At the timing of a shift re-set the summing registers then enter ACQUIRE and CALCULATE modes. Initially the user program must write an approximate mean into Zm + 10. Scale the output value of standard deviation x 10 representing one decimal place. At the end of a shift turn control modes off.

The following control rungs are a possible implementation of the requirements above.


START
A3.0
RUN
A3.1
EOS
A3.15
RUN
G99.0
AND
0
CNTRL
OUT
G101
RUN
G99.0
OBEY
BLOCK
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**********START OF BLOCK**********
AND
@13
CNTRL
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N
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NZERO
G99.2
AND
W108
G99.2
OUT
W108
APPXMN
NZERO
AND
W110
G99.2
G99.1
OBEY
BLOCK
*
**********START OF BLOCK**********
AND
@14
CNTRL1
OUT
G101
SETUP
G99.1
**********END OF BLOCK**********
**********END OF BLOCK**********
XI
G100
STDDEV
T18
VALUE
W100
STNDVN
OUT
W111
CNTRL
G101

7. Theory

Derivation of form of standard deviation equation used in calculation.

The standard form of the equation for the variance δ^2 is:

$$\delta^2 = \frac{(x_i - \bar{x})^2}{n-1}$$

Expanding the bracketed term:

$$\Sigma(x_i - \bar{x})^2 = \Sigma(x_i^2 - 2\bar{x}x_i + \bar{x}^2)$$

$$= \Sigma(x_i^2 - 2\bar{x}x_i) + n\bar{x}^2$$

Substituting for x_i in the middle term using

$x = \Sigma x_i/n$ reduces to:

$$\Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - n\bar{x}^2 \quad (1)$$

Thus variance is:

$$\delta^2 = \frac{1}{n-1} (\Sigma x_i^2 - n\bar{x}^2) \quad (2)$$

Instead of the true mean, x, in the above expression, use the approximate mean, A.

$$\delta^2 \text{ approx} = \frac{1}{n-1} (\Sigma x_i^2 - nA^2) \quad (3)$$

Define the error, E, to be added to δ^2 approx. so that:

$$\delta^2 = \delta^2 \text{approx} + E \qquad (4)$$

Subtracting (3) from (2) yields:

$$E = \frac{n(A^2 - \bar{x}^2)}{n-1}$$

Now,

$$\sum_{i=1}^{n} x_i^2 - nA^2 = \sum_{i=1}^{n} (x_i^2 - A^2)$$

Hence:

$$\delta^2 = \frac{1}{n-1} \left(\sum (x_i^2 - A^2) + n(A^2 - \bar{x}^2) \right)$$

Yielding the form of standard deviation:

$$\delta = \sqrt{\delta^2}$$

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